

UK JUNIOR MATHEMATICAL CHALLENGE

TUESDAY 27TH APRIL 1999

Organised by the **United Kingdom Mathematics Trust**
from the **School of Mathematics, University of Leeds**



SOLUTIONS LEAFLET

This is the first solutions leaflet which has been provided for a JMC. It is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. C Thirty-four hundredths = 0.34.
2. C 200ml is approximately one third of a pint or just under two thirds of the volume of a typical can of soft drink (330ml).
3. A The hour hand will be half way between the 'one' and the 'two' whilst the minute hand will still point to the 'six'.
4. C 7000000 is exactly divisible by 7, so we need only to calculate the remainder when 10 is divided by 7.
5. A There must be seven 26p stamps to give a total ending in 2.
 $7 \times 26p = \text{£}1.82$ and therefore there is only one 20p stamp.
6. B $19 + 99 + 19 \times 99 = 99 + 19 \times 1 + 19 \times 99 = 99 + 19 \times 100 = 99 + 1900$
or $19 + 99 + 19 \times 99 = 19 + 99 \times 1 + 99 \times 19 = 19 + 99 \times 20 = 19 + 1980$
7. D There are eight people, each of whom buys seven eggs each.
8. E Total debt = $\text{£}55 \times 55 = \text{£}3025$. $\text{£}50 \times 50 + \text{£}5 \times 5 = \text{£}2525$. I have $\text{£}500$ too little.
9. C Only 21 of the 22 slices may be used and these will make seven 'double-decker' sandwiches. Each sandwich requires four sides of bread to be buttered.
10. A Laa-laa's luggage is 15kg overweight. The limit above which a charge is made is therefore 35kg. Po's luggage weighs less than this and therefore she will not be charged.
11. C $2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 = 100$.
(By coincidence, the number of prime numbers less than 100 is 25.)
12. E There are 3 routes from S to U and 2 from U to V . The number of different routes from S to V is therefore $3 \times 2 = 6$. Each of these may be followed by any one of three different routes from V to T , making a total of $6 \times 3 = 18$ routes in all.
13. D $\angle PMR = 110^\circ$ (adjacent angles on a straight line); $\angle PRM = 50^\circ$ (angle sum of a triangle) $\angle PRS = 130^\circ$ (adjacent angles on a straight line).
(A shorter method uses the exterior angle theorem:
 $\angle PRS = \angle MPR + \angle PMR$.)
14. D Let Ross drink x ml. Then Rachel drinks $\frac{3}{2}x$ ml. $\frac{3}{2}x + x = 750 \Rightarrow \frac{5}{2}x = 750 \Rightarrow x = 300$. Rachel, therefore, drinks 450ml.
15. C The distance the snail moves East = $2 - 4 + 6 - 8 + 10 = 6$.
The distance the snail moves North = $1 - 3 + 5 - 7 + 9 = 5$.
16. D $-1 + 4 = 3$; $2 + 2 = 4$; $1 + 6 = 5$; $2 + 6 = 8$.
17. E A test for divisibility by 11 is to add alternate digits:

$$1 + 3 + * + 7 = 11 + *; \quad 2 + 4 + 6 + 8 = 20.$$

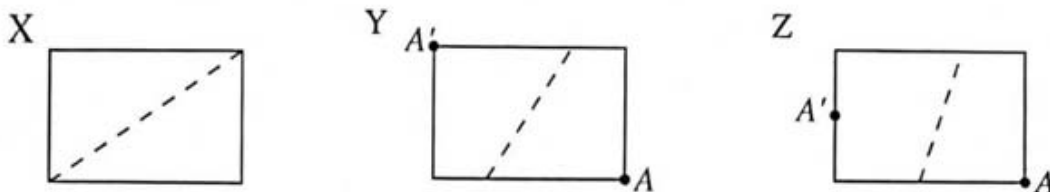
If the original number is a multiple of 11 then these two totals will be the same or will differ by a multiple of 11. In this case, $11 + * = 20 \Rightarrow * = 9$.

Or, you can solve it without knowing a rule as follows:-

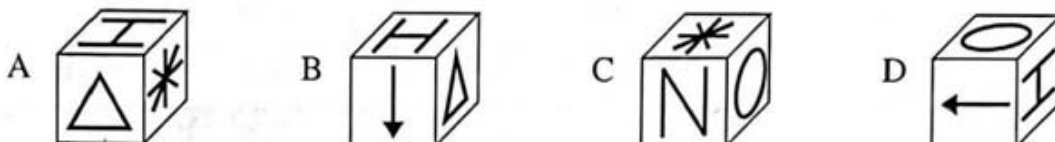
$1234*678 = 12340678 + 1000* = (11 \times 1121879 + 9) + 11 \times 90* + 10*$
and hence is divisible by 11 $\Leftrightarrow 10* + 9$ is divisible by 11. So $* = 9$.

18. B $6729 \times 2 = 13458$ and therefore $\frac{6729}{13458}$ is equal to one half and uses all the digits from 1 to 9 inclusive.

19. E The required fold lines are shown (A folds to A').



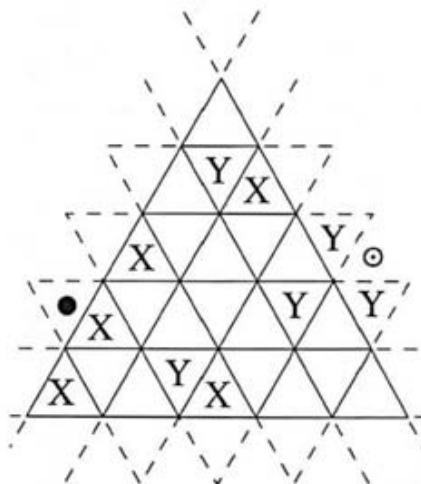
20. E With the front face as shown, the cubes A – D would appear as shown:



21. D $\frac{6}{7}$ of Granny's age is 84. Her age, therefore, is $84 \times \frac{7}{6} = 98$.

22. D The hands are at right angles to each other twice in every hour except that the only time between 2.00 and 3.00 is just after 2.27 and the only time between 8.00 and 9.00 is just after 8.27.

23. B In the diagram the triangles labelled X can be reached from \bullet in one 'move'; those labelled Y are triangles which are one 'move' away from \odot . This shows that it is not possible to go from \bullet to \odot in one 'move', nor in two 'moves' since none of the triangles are labelled both X and Y. However, some of the triangles labelled Y may be reached from a triangle labelled X in one 'move' and therefore it is possible to complete the required task in three 'moves'.



24. A Let the time which Boris takes to run up five steps be T . Therefore:

Boris takes $\frac{99}{5}T$ to run up the 99 steps;

Spike takes $\frac{78}{4}T$ to run up the $(99 - 21)$ steps;

Percival takes $\frac{61}{3}T$ to run up the $(99 - 38)$ steps.

$\frac{78}{4} < \frac{99}{5} < \frac{61}{3}$ and therefore the order is Spike, Boris, Percival.

25. D The multiplication sum is 43×29 .

There cannot be a carry from the units column into the tens column in the final addition and this tells us we have 860 on the fourth line. This must come from 43 multiplied by a number between 20 and 29 inclusive. The only single digit which multiplies 43 to give a three digit number with 8 as the middle digit (third row) is 9 and therefore we have

$$\begin{array}{r} 43 \\ \times 29 \\ \hline 387 \\ 860 \\ \hline 1247 \end{array}$$